Noise Propagation around a Thin Half-Plane *

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Summary

The detailed behavior of Macdonald's rigorous solution for a spherical sound diffraction by a half-plane are clarified by numerical calculation, and it is shown that all experimental results are in a very good agreement with the rigorous solution no matter where a sound source and an observation point are located, and that the approximate formula of the rigorous solution given accurately by Bowman and Senior has little approximation error.

Further a new simple approximate formula of the rigorous solution is presented and a practical method based on the new approximate formula is proposed for the prediction of the propagation of noise, by which the detailed peak-dip due to interference between several waves are not drawn in the sound pressure level distributions but only monotonous gradual variations are drawn.

Schallausbreitung an einer dünnen Halbebene

Zusammenfassung

Das genaue Verhalten der Macdonaldschen, strengen Lösung für sphärische Schallbeugung an einer Halbebene wird durch numerische Berechnung dargestellt. Es wird gezeigt, daß alle experimentellen Ergebnisse ausgezeichnet mit der strengen Lösung übereinstimmen — unabhängig davon, wo sich Schallquelle und Beobachtungspunkt befinden — und daß die von Bowman und Senior angegebene Näherungsformel für die strenge Lösung recht genau ist und wenig Näherungsfehler enthält.

Ferner wird eine neue, einfache Näherungsformel für die strenge Lösung angegeben und es wird eine auf der neuen Näherungsformel basierende, praktische Methode für die Berechnung der Schallfortpflanzung vorgeschlagen. Bei dieser Methode wird der durch Interferenz zwischen verschiedenen Wellen hervorgerufene Spitzenabfall nicht detailliert in die Schalldruckpegelverteilung einbezogen, sondern es werden nur monotone, gleichmäßige Schwankungen berücksichtigt.

Propagation du bruit dans un demi-plan mince

Sommaire

Macdonald a donné la solution rigoureuse de la diffraction d'ondes sphériques par un demi-plan: le comportement de cette solution est ici clarifié par le calcul numérique, et l'on montre qu'elle est en accord avec les observations, quelles que soient les positions de la source et du récepteur. On montre également que la formule approchée de Bowman et Senior n'introduit que de faibles erreurs. Enfin, on propose une nouvelle formule approchée simple, ainsi qu'une méthode pratique simple pour prédire la propagation du bruit, qui néglige les pointes abruptes dues à l'interférence de certaines ondes, et ne met en évidence que des variations graduelles et monotones.

1. Introduction

In predicting and controlling the propagation of noise in the open air, it is very important to take account of the shape of a building that is the noise source, the cross section of railway track and road, as well as barriers and surrounding buildings. Then, the theory of the free-field diffraction of a spherical sound wave by a thin half-plane is basically necessary.

Fresnel's zone construction theory, for which Huygens' Principle is applied, and Kirchhoff's approximate solution obtained by mathematically deriving a rigorous integral formula for Huygens' Principle are well known as classical diffraction theories [1] and are widely used until now, where it is supposed that the linear dimensions of an opening are large compared with the wave-length and the diffracting plane is perfectly absorbent.

On the other hand, in contrast to these approximate diffraction theories, a rigorous solution based on rigorous boundary conditions was obtained by Sommerfeld [2] for the incidence of a plane wave on a thin half-plane. Further, based on this solution, Carslaw [3] obtained a solution for the incidence of a spherical wave. Moreover, Macdonald [4] provided this in the form of an arranged rigorous solution.

However, the rigorous solution is hardly used in calculating noise reduction in the design of barriers

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etc., and instead, Maekawa's experimental curve [5] is widely used. But the range of application of the experimental curve is limited, since it is obtained by experiments which satisfy Kirchhoff's approximate conditions. On the other hand, in considering the noise propagation etc. from the outer wall surfaces or roof surfaces of a building, it is clearly a very important and fundamental model, that a sound source is located on the surface of a half-plane, which does not satisfy the above conditions. Further, we must predict the propagation of noise when a sound source and an observation point are on the same side of the plane, as equally well as when they are on opposite sides of the wall. Therefore, it is necessary to treat the propagation of noise in the whole region.

From the above points of view, in this paper it is to be noted that Macdonald's rigorous solution can be applied no matter where sound source and observation point are located, and the surprisingly unknown characteristics are clarified by numerical integration etc. Further, detailed experiments are shown for the observation point in the whole region including the cases where a sound source is on the surface of a half-plane. From these results, it is shown that the use of rigorous solution, or the approximate formula given accurately by Bowman and Senior [6], are necessary for calculating the propagation of sound with an extremely high accuracy.

However generally for predicting the propagation of noise in practical cases, it is not necessary to draw the detailed peak-dip of SPL (sound pressure level) distributions, which is due to interference between several waves. Therefore in the interfered region the direct use of the rigorous solution or the approximate formula is not appropriate for practical cases. So firstly a new simple approximate formula of the rigorous solution is presented, which agrees extremely well with Bowman and Senior's approximate formula and needs only half the computer calculation time compared with Bowman and Senior's formula. Next a practical method based on the new approximate formula is proposed for predicting the propagation of noise, by which the detailed peak-dip due to interference between several waves are not drawn in the SPL distributions but only monotonous gradual variations are drawn. Further the practical method is comparable with the rigorous solution or with the experimental results.

2. Rigorous solution

As shown in Fig. 1, the infinitely thin, acoustically hard half-plane is defined in terms of the rectangular coordinates by $0<x$, $y=0$ or in terms of the cylindrical coordinates by $\theta=0$ and $\theta=2\pi$ and a point sound source is located at $P_0(r_0, \theta_0, z_0)$ where it is assumed that $0 \leqq \theta_0 \leqq \pi$. The velocity potential $\Phi=\varphi\, \mathrm{e}^{-\mathrm{i}\omega t}$ by its spherical waves at an arbitrary point $P(r, \theta, z)$ within all region is given by the following equation [2], [3], [4] when the time factor is eliminated and the incidence spherical wave is

$$\begin{aligned} \varphi_\mathrm{i} &= \mathrm{e}^{\mathrm{i}kR}/R\,, \\ \varphi &= U(\theta_0) + U(-\theta_0) = \\ &= \mathrm{i}\,k \int_{-\tau_R}^{\infty} \frac{H_1^{(1)}(\tau^2+kR)}{\sqrt{\tau^2+2kR}}\,\mathrm{d}\tau + \\ &\quad + \mathrm{i}\,k \int_{-\tau_{R'}}^{\infty} \frac{H_1^{(1)}(\tau^2+kR')}{\sqrt{\tau^2+2kR'}}\,\mathrm{d}\tau\,, \end{aligned} \tag{1}$$

where

$$\begin{aligned} \tau_R &= \mathrm{sgn}(\pi-\theta+\theta_0)\sqrt{kR_1-kR}\,, \\ \tau_R &= \mathrm{sgn}(\pi-\theta-\theta_0)\sqrt{kR_1-kR'}\,, \\ R &= \sqrt{r^2+r_0^2+(z-z_0)^2-2rr_0\cos(\theta-\theta_0)} \\ R' &= \sqrt{r^2+r_0^2+(z-z_0)^2-2rr_0\cos(\theta+\theta_0)} \\ R_1 &= \sqrt{(r+r_0)^2+(z-z_0)^2} \\ \mathrm{sgn}(\Theta) &= \begin{cases} +1 & \text{for} \quad \Theta>0 \\ -1 & \text{for} \quad \Theta \leqq 0\,. \end{cases} \end{aligned}$$

Here, $H_1^{(1)}$ is the first order Hankel function of the first kind and R, R' are respectively the distances from the observation point P to the real sound source and the image source. R_1 is the shortest distance to the point P from point P_0 or P_0' in stepping over the edge of the half-plane. We can easily understand that $U(\theta_0)$ means the velocity potential due to the real sound source while $U(-\theta_0)$ means that of the image source.

Now if the excess sound attenuation ATT by a screen is defined by

$$\mathrm{ATT} = -20\log|\varphi/\varphi_\mathrm{i}|\ \mathrm{dB} \tag{2}$$

we obtain

$$\mathrm{ATT} = -20\log\left| kR\left\{ \int_{-\tau_R}^{\infty} \frac{H_1^{(1)}(\tau^2+kR)}{\sqrt{\tau^2+2kR}}\,\mathrm{d}\tau + \int_{-\tau_{R'}}^{\infty} \frac{H_1^{(1)}(\tau^2+kR')}{\sqrt{\tau^2+2kR'}}\,\mathrm{d}\tau \right\}\right|\ \mathrm{dB}\,. \tag{3}$$

Since τ_R and $\tau_{R'}$ contains kR_1, the attenuation is represented with kR, kR' and kR_1 as variables. Now if R, R' and R_1 are normalized by a half wave-length as

$$\begin{aligned} R_N &= 2R/\lambda = kR/\pi \\ R'_N &= 2R'/\lambda = kR'/\pi \\ R_{1N} &= 2R_1/\lambda = kR_1/\pi \end{aligned} \tag{4}$$

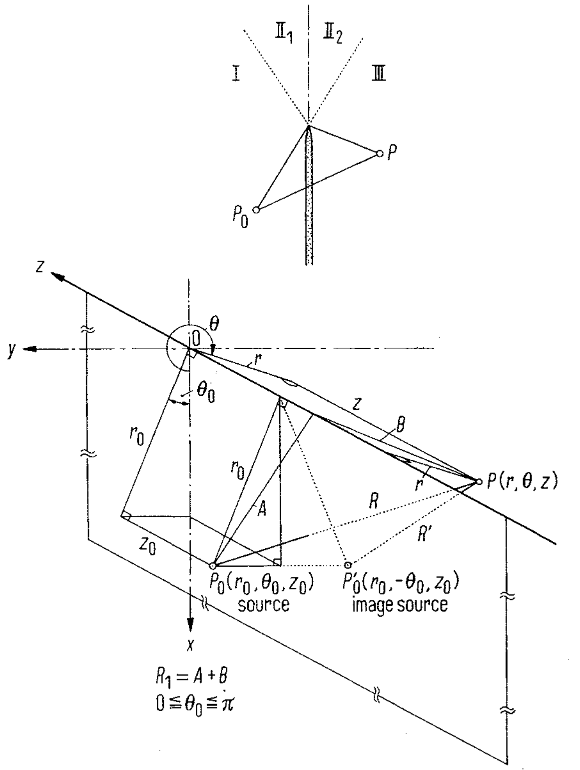

Fig. 1. Geometry of a point source, an observation point and a half-plane.

the attenuation by a screen can be obtained by these three normalized distance parameters and the region I, II or III as shown in Fig. 1, in which the observation point is located. Further, so called Fresnel's zone number N means the normalized parameter δ_N of the path difference δ as given in the following equation.

$$\begin{aligned} N = \delta_N &= \frac{2\delta}{\lambda} = \\ &= -\operatorname{sgn}(\pi - \theta + \theta_0)(R_{1N} - R_N), \\ N' = \delta'_N &= \frac{2\delta'}{\lambda} = \\ &= -\operatorname{sgn}(\pi - \theta - \theta_0)(R_{1N} - R'_N). \end{aligned} \tag{5}$$

Now in eq. (1) if

$$\pi \ll k R_1 \tag{6}$$

by using complex Fresnel Integral

$$F(a) = \int_a^\infty e^{i\tau^2}\, d\tau$$

the following approximate form is correctly derived by Bowman and Senior [6].

$$\begin{aligned} U(\theta_0) &= U_g(\theta_0) + U_d(\theta_0) \cong \\ &\cong \eta(\pi - \theta + \theta_0)\frac{e^{ikR}}{R} + \operatorname{sgn}(\pi - \theta + \theta_0)\, i \times \\ &\times \sqrt{\frac{2}{\pi R_1 (R_1 + R)}} \exp\left(ikR + i\frac{\pi}{4}\right) F(|\tau_R|) \end{aligned} \tag{7}$$

where,

$$\eta(\Theta) = \begin{cases} +1 & \text{for } \Theta > 0 \\ 0 & \text{for } \Theta \leqq 0. \end{cases}$$

According to this, on the boundary of the geometrically illuminated region and the shadowed region, namely $\theta \to \theta_0 \pm \pi$ it besomes,

$$U(\theta_0) \to \frac{e^{ikR}}{2R} \tag{8}$$

and even if no approximate condition $\pi \ll k R_1$, it always agrees with the results of rigorous solution (1).

Now in eq. (7)

$$\text{if} \begin{cases} R \to \infty \text{ or} \\ R_1 \to R \end{cases}, \quad \sqrt{\frac{2}{\pi R_1 (R_1 + R)}} \to \frac{1}{\sqrt{\pi} R}. \tag{9}$$

Therefore,

$$\begin{aligned} U(\theta_0) &\to \eta(\pi - \theta + \theta_0)\frac{e^{ikR}}{R} + \\ &+ \operatorname{sgn}(\pi - \theta + \theta_0)\frac{i \exp\left(ikR + i\frac{\pi}{4}\right)}{\sqrt{\pi} R} F(|\tau_R|) = \\ &= -\Phi_i \frac{i \exp i\left(\frac{\pi}{4}\right)}{\sqrt{\pi}} F(-\tau_R) \end{aligned} \tag{10}$$

is obtained and this agrees with Kirchhoff's approximate theoretical formula with an improved accuracy in approximation [5]. In other words it is said that Kirchhohff's approximate formula is based on two approximate conditions as mentioned below.

Approximate conditions:

1. Contribution by the image sound source is neglected.

2. The observation point is located either close to the boundary $\theta \cong \theta_0 + \pi$ of the geometrical optics regions for the real sound source or sufficiently away from the source $R \to \infty$.

3. Results of numerical calculation by rigorous solution

Now we introduce the attenuation ATT_{θ_0} by a screen where the contribution of the image source is neglected and defined by

$$\mathrm{ATT}_{\theta_0} = -20 \log \left| \frac{U(\theta_0)}{\varphi_\mathrm{i}} \right| \mathrm{dB}. \tag{11}$$

First the behaviour of ATT_{θ_0} which is taken into account with only the real sound source is considered and then the deviation from ATT_{θ_0} to ATT as a result of the contribution of the image source.

Results of the numerical calculation of ATT_{θ_0} are shown in Fig. 2. The attenuation by a half-plane varies not only with so-called Fresnel Zone Number N value, but also widely with R_N as a parameter, and as mentioned in chapter 2, it agrees with Kirchhoff's approximate value when $R_\mathrm{N} \to \infty$. When $N = 0$, i.e. on the shadow boundary, always it becomes $\mathrm{ATT}_{\theta_0} = 6$ dB, as shown in eq. (8). Further, the approximate formula (7) provided by Bowman and Senior has an extremely high accuracy

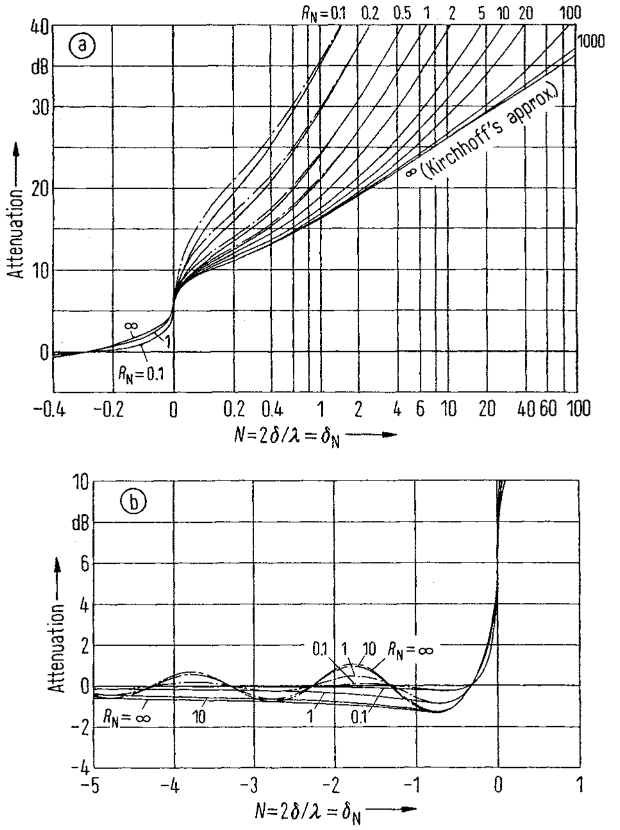

Fig. 2. Calculated sound attenuation by a screen ATT_{θ_0} where the image source contribution is neglected.
(a) $-0{,}4 \leqq N$.
——— Practical eq. (16), approximate solution by Bowman and Senior,
—·— rigorous solution by Macdonald.
(b) $N \leqq 0$.
——— Practical eq. (16),
—·— rigorous solution by Macdonald, approximate solution by Bowman and Senior.

of approximation, namely the level difference from the rigorous solution (1) is smaller than 0.5 dB only if $0.5 < R_\mathrm{1N}$, i.e. $\lambda/4 < R_1$.

As mentioned in chapter 2, the so-called attenuation ATT by screen, including the contribution of image sound source, varies not only with the normalized distance parameters R_N, R_1N, but also with R'_N. As it's maximum deviation from ATT_{θ_0} is 6 dB, the contribution of image source is extremely large. For $0 < \theta_0 < \pi$ and $\pi \leqq \theta \leqq 2\pi$ ($R'_\mathrm{N} \leqq R_\mathrm{N}$) i.e. when the source is located away from a half-plane and the observation point is on the opposite side of the plane we get

$$\mathrm{ATT}_{\theta_0} - 6 \leqq \mathrm{ATT} \leqq \mathrm{ATT}_{\theta_0}\,\mathrm{dB} \quad \text{for} \quad 0 \leqq \mathrm{ATT}. \tag{12}$$

The behaviour that ATT deviates from ATT_{θ_0} by the contribution of the image source is shown in Fig. 3 for various R_N values. The deviation from ATT_{θ_0} is minimum when $R'_\mathrm{N} = 0$, i.e. the observation point is at the position of image source and in other words ATT becomes maximum for all R_N, R_1N values. On the other hand, when $R_\mathrm{N} = R'_\mathrm{N}$, deviation from ATT becomes maximum and ATT becomes minimum for all R_N, R_1N values. At first, along the extension of a half-plane, i.e. when $\theta = \pi$, we get

$$\mathrm{ATT} = 0\,\mathrm{dB} \quad \because \quad \varphi = \varphi_\mathrm{i} + \varphi_\mathrm{s} = \varphi_\mathrm{i}. \tag{13}$$

This is due to the fact that the scattered wave $\varphi_\mathrm{s} = 0$ for $\theta = \pi$ as the scattered field is antisymmetric with respect to plane $y = 0$, since φ_s is a sound wave radiated by a plane vibrating as $[+\partial\varphi_\mathrm{i}/\partial y]_{y=0}^{x \lessgtr 0}$. When the position of source and the observation point is reversed, i.e. the sound source is located along the extension of the plane, eq. (13) remains valid no matter where the observation point is located, as seen from reciprocal theorem, since spherical waves are propagated in the whole region without depending on the existence of a plane. Secondly, when the observating point exists on the surface of a plane, i.e. for $\theta = 0$ or 2π, ATT for an arbitrary position of source becomes

$$\mathrm{ATT} = \mathrm{ATT}_{\theta_0} - 6\,\mathrm{dB} \quad \because \quad U(\theta_0) = U(-\theta_0) \tag{14}$$

in comparison with ATT_{θ_0} shown in Fig. 2. Eq. (14) is also valid, when the conditions are reversed, i.e., for the sound source on the surface of a half-plane and the arbitrary point of the observation.

Since Maekawa's experimental curve, which is drawn in Fig. 3 for comparison, has only N as a variable, it largely differs from the results of rigorous solution which has three distance parameters R_N, R'_N and R_1N.

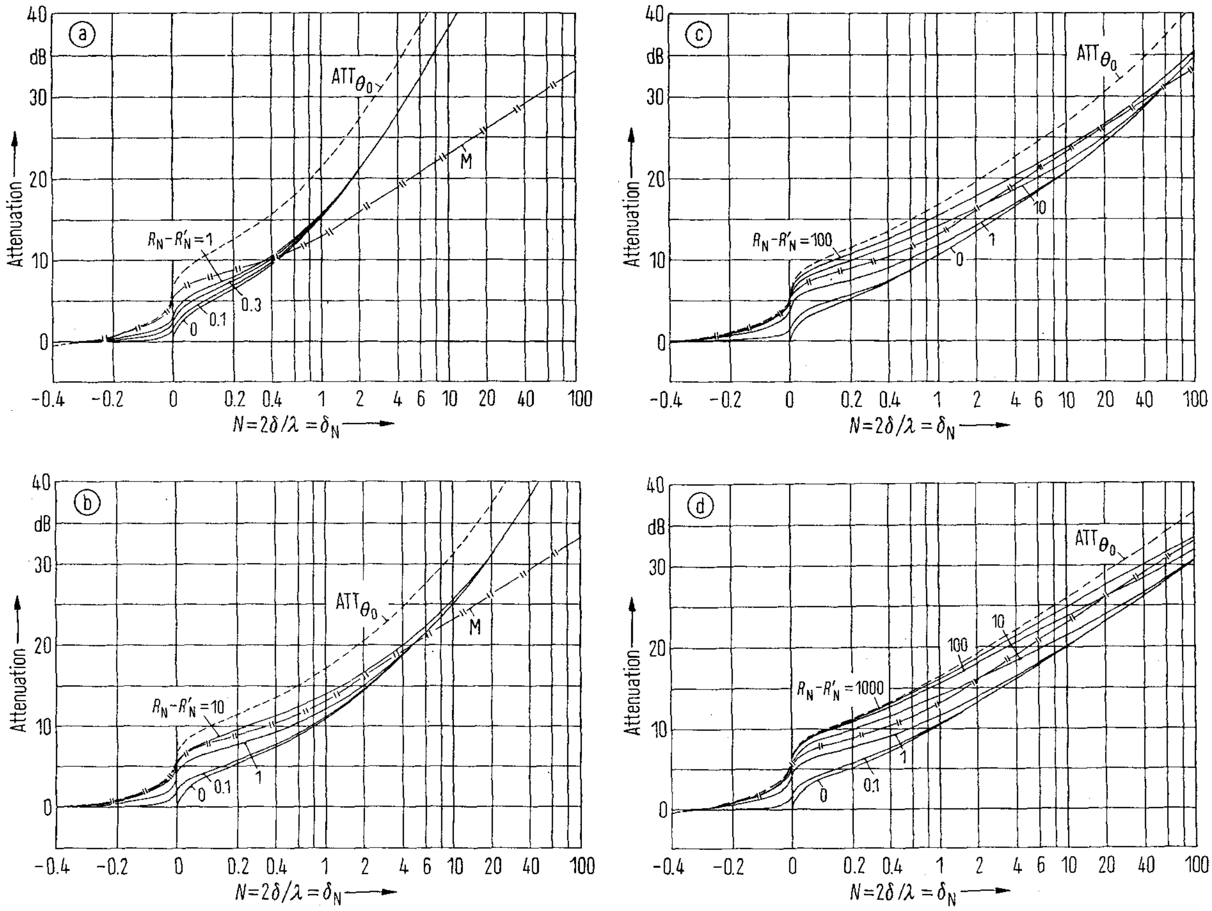

Fig. 3. Calculated sound attenuation ATT where the image source contribution is considered.
$0 < \theta_0 \leqq \pi$, $\pi \leqq \theta \leqq 2\pi$ ($R'_N \leqq R_N$).
(a) $R_N = 1$, (b) $R_N = 10$, (c) $R_N = 100$, (d) $R_N = 1000$.
M = Maekawa's experimental curve.

4. Experimental method

As shown in Fig. 4, a barrier made by sandwitching a lead sheet of thickness 0.5 mm with 12 mm thick plywoods, was set up as a half-plane in an anechoic room which is 4.8 m × 6 m × 4.4 m internally large. The barrier was extended up to the concrete side walls of the room and the joints between the barrier and the wall or floor were hard-pressed with plastic clay. The surface of the barrier was painted and the top was wedged to an angle of 30° over which a aluminium foil was stuck.

In order to minimize the measurement errors due to thickness of barrier wall, shape of edge and the directivity characteristics of speaker or microphone, etc., the measurement should be made for frequencies as low as possible. Further, since inverse square law is sufficiently applicable in the anechoic room for a frequency greater than 400 Hz, 1/3 octave band noise of middle frequency 500 Hz and 1 kHz were selected as the measuring sound. The speaker used was the driver unit YL-D 5500 P for horn speakers and the directivity characteristics are extremely good with

500 Hz: − 0.5 dB
1 kHz: − 1.0 dB

in the direction normal to front face axis.

The position of sound source was selected at a total of eight points as shown in Fig. 4. They are sound sources $S_{1,2,3,4}$ on the surface of barrier and $S_{11,21,31,13}$ located away from the barrier. Receiving planes selected were planes $z = 0$, 1.6 m and the measuring region was taken as

$$-1.6\ \text{m} \leqq x \leqq 1.02\ \text{m}$$
$$-2.1\ \text{m} \leqq y \leqq 2.1\ \text{m}$$

for both planes. Continuous recordings of SPL were made by moving a 1 inch condenser microphone automatically.

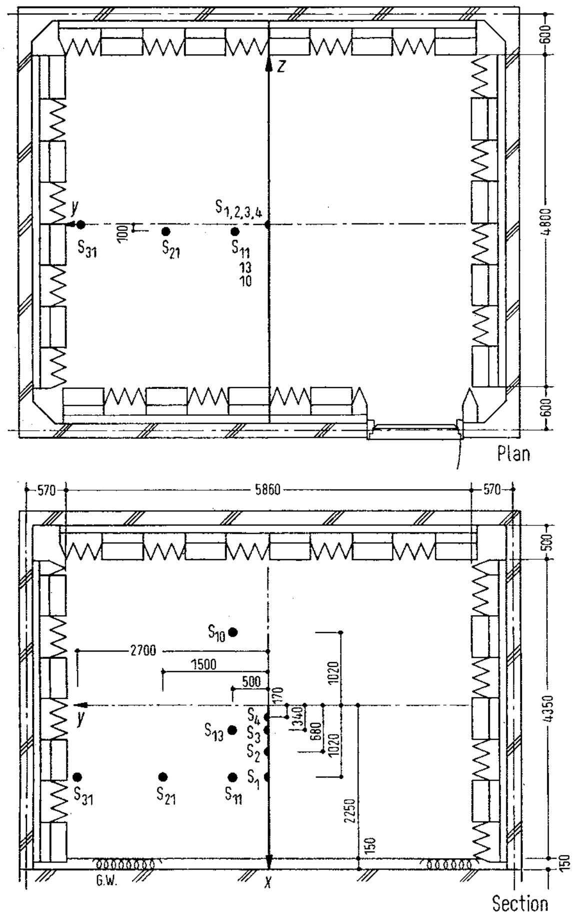

Fig. 4. A half-plane set up in an anechoic room. (All dimensions in mm.)

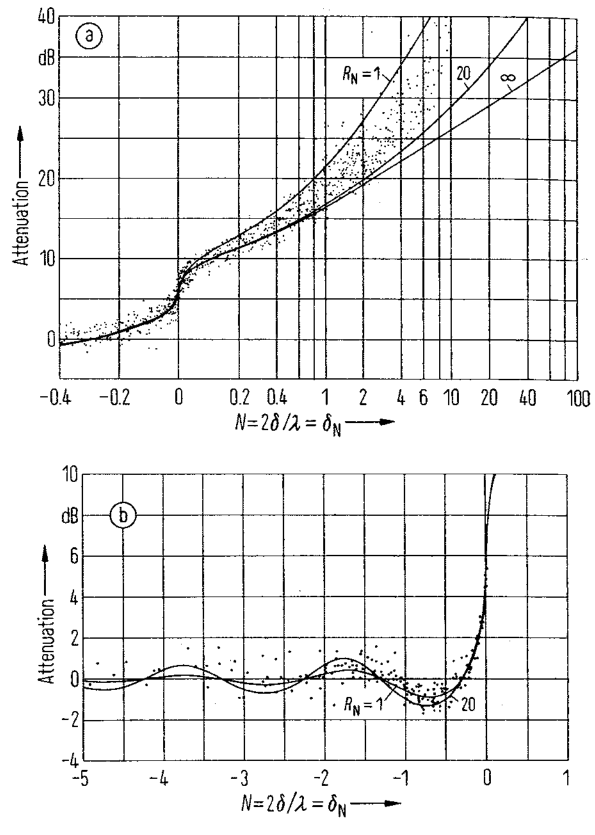

Fig. 5. Measured sound attenuation by a screen ATT by the sources $S_{1,2,3,4}$.
——— Rigorous solution by Macdonald.
(a) $-0{,}4 \leq N$.
(b) $N \leq 0$.

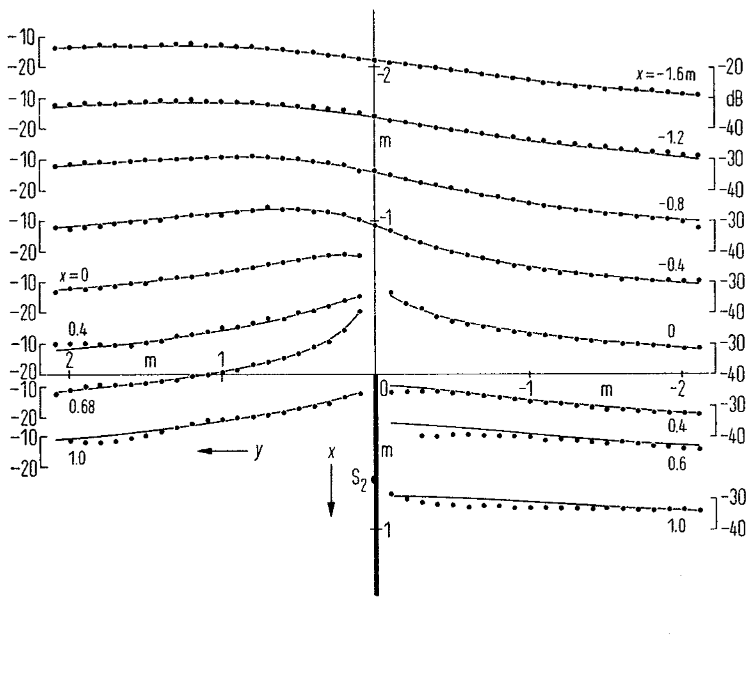

Fig. 6. Comparison between calculated SPL distributions by Macdonald's rigorous solution (———) and the measured one (● ● ●) for S_2, 500 Hz, $z = 0$. 0 dB = PwL.

5. Experimental results

Since the real sound source and image source coincides when the sound source lies on the surface of a half-plane, the attenuation ATT_{θ_0} by a screen neglecting the contribution of image source could be directly measured from eq. (14). As shown in Fig. 5, the measured ATT_{θ_0} for sources $S_{1,2,3,4}$ agree very well with the rigorous solution, for R_N is found to be between 1 and 20 according to the positions of sound sources and the points of observation in the experiment. Further, SPL distributions along each x-straight line on plane $z=0$ for the source S_2 with 500 Hz band noise are shown in Fig. 6, while the equi-SPL contours are shown in Fig. 7. From these figures, it is clear that the measured results agree extremely well with the rigorous solution.

Next, Fig. 8 shows a comparison of measured ATT for sound sources $S_{11,21,31,13}$ with the rigorous solution. The curves obtained by rigorous solution show the maximum and minimum values of ATT for each value of N, when R_N and R_N' are within the range of experimental conditions. Measured ATT plotted lie almost completely with in the region surrounded by them. Also SPL distributions along x-straight lines on plane $z=0$ for S_{11} are shown in Fig. 9 and equi-SPL contours are shown in Fig. 10. It can be seen from these figures that although measured values scatter slightly in the region I in Fig. 1 where the image sound source could be seen, in the other regions they agree extremely well with the rigorous solution. Further, Maekawa's experimental curve is obtained for the case when the observation point and the sound source are on opposite sides of a half-plane, and it is clear that the values by the curve deviate largely from our experimental results, when the sound source or the observation point is comparatively close to the plane.

6. Practical method for predicting the propagation of noise

Now come back to the approximate representation (7) of Macdonald's rigorous solution (1). From eq. (7) expressed in terms of complex Fresnel integrals, the following numerical approximation

$$U(\theta_0) \simeq \eta(-N)\frac{\mathrm{e}^{\mathrm{i}kR}}{R} + \operatorname{sgn}(-N) \times \times \sqrt{\frac{2}{R_1(R_1+R)}}\, A(R)\, E(R) \quad (15)$$

where

$$A(R) = \begin{cases} \dfrac{1}{2\pi\sqrt{|N|}}\left\{1 - \dfrac{0.08}{(|N|+0.4)^2}\right\} & \text{for } 0.8 \leqq |N| \\ -(C+\mathrm{i}S) & \text{for } |N| < 0.8 \end{cases}$$

$$C = \begin{cases} 0.5 - \sqrt{|N|}/1.68 & \text{for } |N| \leqq 0.1 \\ 1.01(|N|-0.75)^2 - 0.17 & \text{for } 0.1 < |N| < 0.8 \end{cases}$$

$$S = \begin{cases} 0.73\sqrt{|N|}(1-|N|/0.75) & \text{for } |N| \leqq 0.5 \\ 1.16\sqrt{|N|}(1-\sqrt{|N|/0.8}) & \text{for } 0.5 < |N| < 0.8 \end{cases}$$

$$E(R) = \begin{cases} \mathrm{e}^{\mathrm{i}(kR_1 - 0.8\pi)} & \text{for } 0.8 \leqq |N| \\ \mathrm{e}^{\mathrm{i}kR} & \text{for } |N| < 0.8 \end{cases}$$

with a similar result for $U(-\theta_0)$, are established. Level difference between ATT_{θ_0} by the above eq. (15) and that by eq. (7) is smaller than 0.05 dB except 0.15 dB for $N \simeq 0.03$. Therefore we can regard that eq. (15) equals eq. (7) and that the level difference of eq. (15) from the rigorous solution (1) is smaller than 0.5 dB only if $\lambda/4 < R_1$. Eq. (15) needs only a half calculation time in a computer compared with eq. (7).

Generally for predicting the propagation of noise, it is not necessary to draw the detailed peak-dip of SPL distributions which is due to interference between the geometrical optics wave U_g and the diffracted wave U_d or between the wave $U(\theta_0)$ from the real sound source and the wave $U(-\theta_0)$ from the image source. Further more, if the SPL distributions are predicted between the peak and dip we might have many risk in designing.

Consequently, it is one of the best methods to draw the SPL distributions by the envelopes of the peaks. Then the plus prediction error becomes minimum.

Now, if it is assumed that $E(R) \to 1$ for all N values in eq. (15), i.e.,

$$U(\theta_0) \to \left\{\frac{\eta(-N)}{R} + \operatorname{sgn}(-N) \times \times \sqrt{\frac{2}{R_1(R_1+R)}}\, A(R)\right\}\mathrm{e}^{\mathrm{i}kR}. \quad (16)$$

ATT_{θ_0} has no peak-dip when $N < -0.8$ as shown in Fig. 2b.

Based on eq. (16), we propose the following practical method no metter where the source is

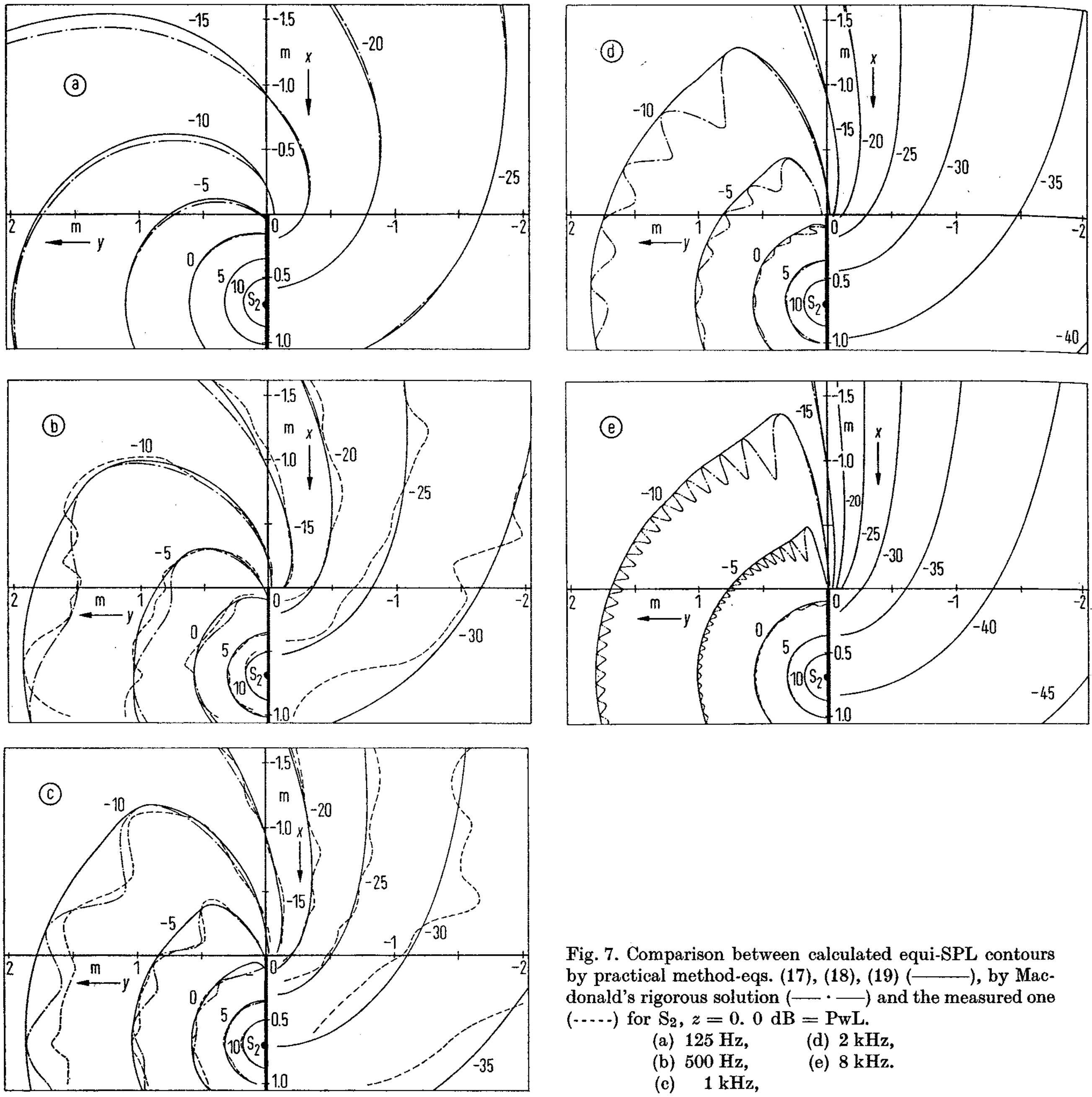

Fig. 7. Comparison between calculated equi-SPL contours by practical method-eqs. (17), (18), (19) (———), by Macdonald's rigorous solution (—— · ——) and the measured one (-----) for S_2, $z = 0$. 0 dB = PwL.

(a) 125 Hz, (d) 2 kHz,
(b) 500 Hz, (e) 8 kHz.
(c) 1 kHz,

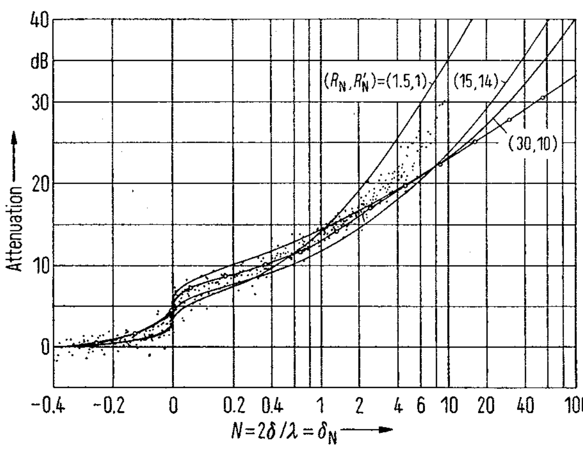

Fig. 8. Measured sound attenuation by a screen ATT by the sources S_{11}, S_{21}, S_{31} and S_{13}.
——— Rigorous solution by Macdonald,
o — o Maekawa's experimental curve.

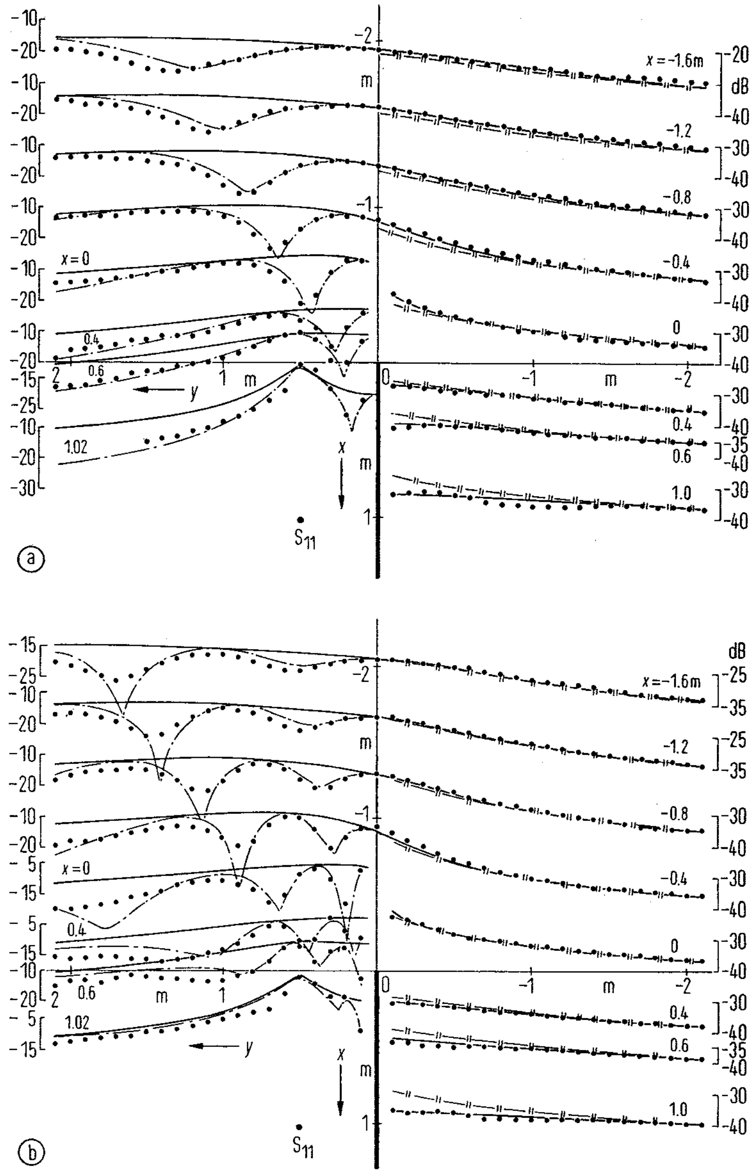

Fig. 9. Comparison between calculated SPL distributions by practical method-eqs. (17), (18), (19) (———), by Macdonald's rigorous solution (— · —), by Maekawa's experimental curve (||—||) and the measured one (● ● ●) for S_{11}, $z = 0$. 0 dB = PwL.
(a) 500 Hz, (b) 1000 Hz.

located.

$$|\varphi| \cong |\psi(R, R')| \qquad \pi \leqq \theta \leqq 2\pi \tag{17}$$

$$|\varphi| \cong |\varphi_i| + |\varphi_s| \cong \frac{1}{R} + |\psi(R', R)| \qquad 0 \leqq \theta < \pi \tag{18}$$

$$\psi(R, R') = \left\{\frac{\eta(-N)}{R} + \operatorname{sgn}(-N)\sqrt{\frac{2}{R_1(R_1+R)}}\,A(R)\right\} e^{ikR} + \operatorname{sgn}(-N')\sqrt{\frac{2}{R_1(R_1+R')}}\,A(R')\,B(R, R') \tag{19}$$

where

$$B(R, R') = \begin{cases} e^{ikR'} & \text{for } |N'| < 0.8 \text{ (then } |N| < 0.8) \\ e^{i(kR_1 - 0.8\pi)} & \text{for } |N'| \geqq 0.8 \text{ and } |N| < 0.8 \\ e^{ikR} & \text{for } |N| \geqq 0.8 \text{ (then } |N'| \geqq 0.8). \end{cases}$$

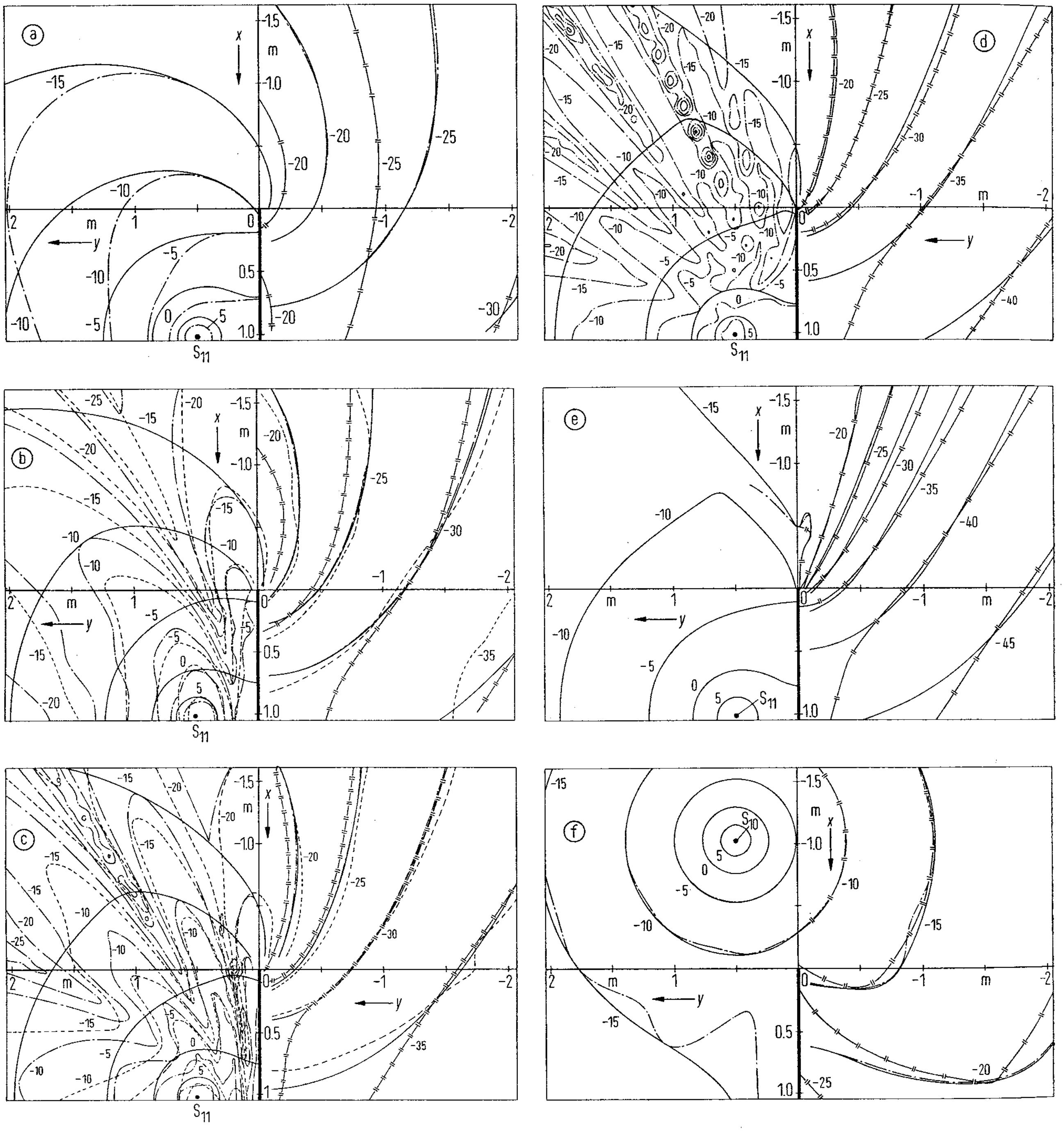

Fig. 10. Comparison between calculated equi-SPL contours by practical method-eqs. (17), (18), (19) (———), by Macdonald's rigorous solution (—— · ——), by Maekawa's experimental curve (||—||) and the measured one (- - - - -) when $z = 0$. 0 dB = PwL.
(a) S_{11}, 125 Hz, (b) S_{11}, 500 Hz, (c) S_{11}, 1 kHz, (d) S_{11}, 2 kHz, (e) S_{11}, 8 kHz, (f) S_{10}, 500 Hz.

Eq. (17) and eq. (18) are continuous at $\theta = \pi$ where the scattered wave $\varphi_s = 0$ and it agrees with the Macdonald's rigorous solution.

As these numerical results are shown in Figs. 7, 9, 10, practical sound pressure level distributions are drawn always by the envelope of the peaks of rigorous one. In the region where are only monotonous gradual variation of SPL by Macdonald's rigorous solution the practical method agrees well with the rigorous one.

However, as an example, in Fig. 10c −10 dB contour by the practical method does not envelop the rigorous one near $x = -0.3$ m, $y = 1.2$ m. It can be understood from the fact that in the equi-SPL contours by the rigorous solution ridges undulated by interference between the geometrical optical wave U_g and the diffracted wave U_d are tracing the place where $U_g(\theta_0)$ i.e. direct wave and $U_g(-\theta_0)$ i.e. reflected wave have the same phase, while in the practical method after the undulated ridges are changed into the ridges which become monotonously low and envelop the peaks, the valleies between the ridges are filled up.

From the above mentioned the proposed practical method based on Macdonald's rigorous solution is convenient for the prediction of the propagation of noise with an extremely high accuracy in many practical cases.

7. Conclusion

The properties of Macdonald's rigorous solution and the approximate formula for a diffracted acoustic field caused by the incidence of a spherical wave on a thin half-plane were clarified by numerical integration etc. Further these were compared with experimental results obtained including the cases when sound source is located on the half-plane and with Maekawa's experimental curve which are widely used in the calculation of noise reduction. As a result, the following were clarified:

1) According to the rigorous solution, sound attenuation by a screen contributed by image source is determined by three normalized distance parameters R_N, R_N' and R_{1N}.

2) Kirchhoff's approximate solution is in agreement with the expression where the contribution of image source is neglected and $R_{1N} \simeq R_N$ in the rigorous solution.

3) The rigorous solution and all of experimental results agree extremely well no matter where the sound source and the observation point are located.

4) Maekawa's experimental curve deviates largely from the experimental results and the rigorous solution when either the source or the observating point is comparatively near the half-plane.

5) The approximate formula of Macdonald's rigorous solution correctly derived by Bowman and Senior has little approximation error which is smaller than 0.5 dB only if $\lambda/4 < R_1$.

Based on the above results, a new simple approximate formula of the rigorous solution, which agrees extremely well with Bowman and Senior's approximate formula and needs only a half calculation time in a computer compared with Bowman and Senior's formula, was presented. Finally a practical method based on the new approximate formula was proposed, by which the detailed peak-dip due to interference between several waves are not drawn in the sound pressure level distribution but only monotonous gradual variations are drawn. From comparison with the rigorous solution or the experimental results it was shown that the practical method is very convenient for the prediction of the propagation of noise with an extremely high accuracy in the design of barriers or factories etc. and that this in turn will help in economical limit design.

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